MATHEMATICS SYLLABUS

Pre-University

Higher 3

Syllabus 9820

Implementation starting with
2021 Pre-University Two Cohort

Ministry of Education
SINGAPORE

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1. Introduction

Nature of Mathematics

Mathematics can be described as a study of the *properties*, *relationships*, *operations*, *algorithms*, and *applications* of numbers and spaces at the very basic levels, and of abstract objects and concepts at the more advanced levels. Mathematical objects and concepts, and related knowledge and methods, are products of insight, logical reasoning and creative thinking, and are often inspired by problems that seek solutions. *Abstractions* are what make mathematics a powerful tool for solving problems. Mathematics provides within itself a language for *representing* and *communicating* the ideas and results of the discipline.

Importance of Learning Mathematics

Mathematics contributes to the developments and understanding in many disciplines and provides the foundation for many of today's innovations and tomorrow's solutions. It is used extensively to model and understand real-world phenomena (e.g. consumer preferences, population growth, and disease outbreak), create lifestyle and engineering products (e.g. animated films, mobile games, and autonomous vehicles), improve productivity, decision-making and security (e.g. business analytics, academic research and market survey, encryption, and recognition technologies).

In Singapore, mathematics education plays an important role in equipping every citizen with the necessary knowledge and skills and the capacities to think logically, critically and analytically to participate and strive in the future economy and society. In particular, for future engineers and scientists who are pushing the frontier of technologies, a strong foundation in mathematics is necessary as many of the Smart Nation initiatives that will impact the quality of lives in the future will depend heavily on computational power and mathematical insights.

Mathematics at the A-Level

There are four syllabuses to cater to the different needs, interests, and abilitites of students:

- H1 Mathematics;
- H2 Mathematics;
- H2 Further Mathematics; and
- H3 Mathematics.

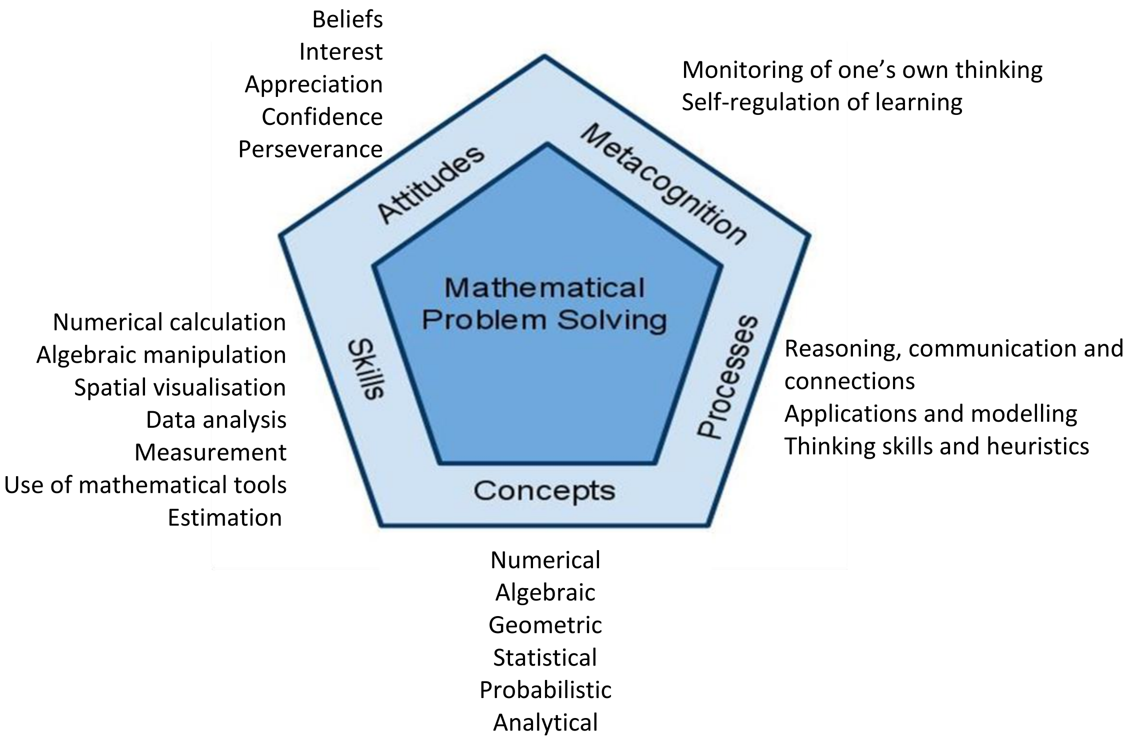
H3 Mathematics is designed for students with the passion and ability in mathematics. It aims to engage students who are keen to specialise in mathematics in solving more challenging problems in mathematics and proving mathematical results. Students will learn useful techniques and results, related to the content in H2 Mathematics, to solve mathematical problems and prove mathematical statements. In the course of learning, students will develop an understanding of the rigour of mathematics and mathematical proof and a deeper insight into the practice, value and beauty of mathematics.

Pre-requisite: H2 Mathematics

Learning mathematics at the A-Level provides students, regardless of the intended course of study at the university, with a useful set of tools and problem solving skills. It also exposes students to a way of thinking that complements other ways of thinking developed through the other disciplines.

Mathematics Curriculum Framework

- *Mathematical Problem Solving*



The central focus of the mathematics curriculum is the development of mathematical problem solving competency. Supporting this focus are five inter-related components – concepts, skills, processes, metacognition and attitudes. The framework sets the direction for and provides guidance in the teaching, learning, and assessment of mathematics.

- *Concepts*

Mathematical concepts can be broadly grouped into *numerical*, *algebraic*, *geometric*, *statistical*, *probabilistic*, and *analytical* concepts. These content categories are connected and interdependent. At different stages of learning and in different syllabuses, the breadth and depth of the content vary.

- *Skills*

Mathematical skills refer to *numerical calculation*, *algebraic manipulation*, *spatial visualisation*, *data analysis*, *measurement*, *use of mathematical tools*, and *estimation*. The skills are specific to mathematics and are important in the learning and application of mathematics. In today's classroom, these skills also include the abilities to use spreadsheets and other software to learn and do mathematics.

- *Processes*

Mathematical processes refer to the process skills involved in the process of acquiring and applying mathematical knowledge. These include *reasoning, communication and connections*, *applications and modelling*, and *thinking skills and heuristics* that are important in mathematics.

Reasoning, communication and connections:

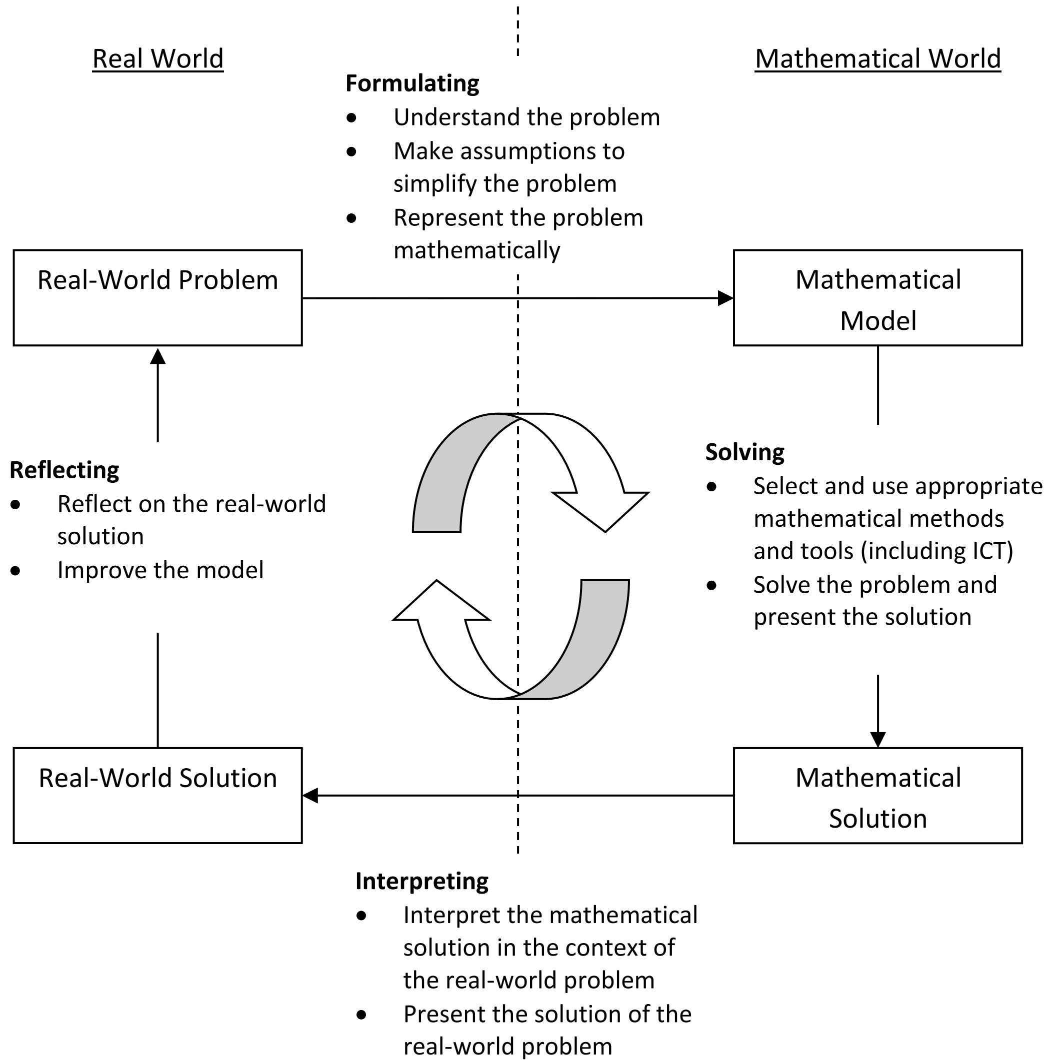
- Mathematical reasoning refers to the ability to analyse mathematical situations and construct logical arguments.
- Communication refers to the ability to use mathematical language to express mathematical ideas and arguments precisely, concisely and logically.
- Connections refer to the ability to see and make linkages among mathematical ideas, between mathematics and other subjects, and between mathematics and the real world.

Applications and modelling allow students to connect mathematics to the real world, enhance understanding of key mathematical concepts and methods, as well as develop mathematical competencies. Mathematical modelling is the process of formulating and improving a mathematical model[1] to represent and solve real-world problems. Through mathematical modelling, students learn to deal with complexity and ambiguity by simplifying and making reasonable assumptions, select and apply appropriate mathematical concepts and skills that are relevant to the problems, and interpret and evaluate the solutions in the context of the real-world problem. [The mathematical modelling process is shown in the diagram on the following page.]

Thinking skills and heuristics are essential for mathematical problem solving. Thinking skills refers to the ability to classify, compare, analyse, identify patterns and relationships, generalise, deduce and visualise. Heuristics are general strategies that students can use to solve non-routine problems. These include using a representation (e.g. drawing a diagram, tabulating), making a guess (e.g. trial and error/ guess and check, making a supposition), walking through the process (e.g. working backwards) and changing the problem (e.g. simplifying the problem, considering special cases).

[1] A mathematical model is a mathematical representation or idealisation of a real-world situation. It can be as complicated as a system of equations or as simple as a geometrical figure. As the word "model" suggests, it shares characteristics of the real-world situation that it seeks to represent.

Mathematical Modelling Process



- *Metacognition*

Metacognition, or thinking about thinking, refers to the awareness of, and the ability to control one's thinking processes, in particular the selection and use of problem-solving strategies. It includes monitoring of one's own thinking, and self-regulation of learning.

- *Attitudes*

Attitudes refer to the affective aspects of mathematics learning such as:

- beliefs about mathematics and its usefulness;
- interest and enjoyment in learning mathematics;
- appreciation of the beauty and power of mathematics;
- confidence in using mathematics; and
- perseverance in solving a problem.

Mathematics and 21CC

Learning of mathematics creates opportunities for students to develop key competencies that are important in the 21st century. As an overarching approach, the A-Level mathematics curriculum supports the development of 21st century competencies (21CC) in the following ways:

1. The content are relevant to the needs of the 21st century. They provide the foundation for learning many of the advanced applications of mathematics that are relevant to today's world.

2. The pedagogies create opportunities for students to think critically, reason logically and communicate effectively, working individually as well as in groups, using ICT tools where appropriate in learning and doing mathematics.

3. The problem contexts raise students' awareness of local and global issues around them. For example, problems set around population issues and health issues can help students understand the challenges faced by Singapore and those around the world.

SECTION 2:
H3 MATHEMATICS SYLLABUS

2. H3 MATHEMATICS SYLLABUS (FROM 2021)

Preamble

Mathematicians work with precise definitions, make conjectures, prove new results and solve problems. They are concerned with the properties of mathematical objects and the applications of abstract ideas and models to solve problems. Mathematical truths and solutions come from rigorously constructed arguments called proofs and mathematically sound procedures and steps. The work of mathematicians has impact in different fields, far beyond just sciences and engineering.

H3 Mathematics provides students, who intend to pursue mathematics at the university, with an insight into the practice of a mathematician. It equips students with the knowledge and skills to understand and write mathematical statements, proofs and solutions, and the techniques and results that come in helpful in their work. Students will develop these competencies through proving mathematical results and solving non-routine mathematical problems in the course of the learning.

Syllabus Aims

The aims of H3 Mathematics are to enable students to:

(a) acquire advanced problem-solving skills and methods of proof by learning useful mathematical results and techniques to solve non-routine problems and prove statements;

(b) develop rigour in mathematical argument and precision in the use of mathematical language through the writing and evaluation of mathematical proofs and solutions; and

(c) experience and appreciate the practice, value and rigour of mathematics as a discipline.

Content Description

Knowledge of the content of H2 Mathematics is assumed.

Students will learn to prove properties and results, and solve non-routine problems involving:

(1) H2 Mathematics content areas

- **(a) Functions**, e.g. graphs, symmetries, derivatives, integrals, differential equations, limiting behaviours, bounds
- **(b) Sequences and series**, e.g. general terms, sum, limiting behaviours, bounds

The examples in (a) and (b) illustrate some types of problems that are based on content in H2 Mathematics.

(2) Additional content areas

(a) Inequalities: AM-GM inequality, Cauchy-Schwarz inequality, triangle inequality
(b) Numbers: primes, coprimes, divisibility, greatest common divisor, division algorithm, congruences and modular arithmetic
(c) Counting: distribution problems, Stirling numbers of the second kind, recurrence equations, bijection principle, principle of inclusion and exclusion.

The above define the expected scope of content knowledge that may be assessed.

Notwithstanding the content areas defined above, students will also prove results and solve problems outside these defined areas or at the intersection of two or more such areas using their ability to understand and apply given definitions or results.

Mathematical Skills

Students are expected to apply the following skills:

Skills	**Examples**
a) Communicate mathematical ideas using mathematical language	• Terms such as "Definition" and "Theorem" • Conditional statements (such as "if *P* then *Q*" and "*P* if and only if *Q*") • Necessary and sufficient conditions • Existential and universal quantifiers (such as "there exists", "for each", "for all") • Logical connectives (such as "and", "or", "not", "implies") • Converse, inverse, contrapositive and negation of statements • Set notation and language
b) Develop and critically evaluate mathematical arguments using mathematical reasoning principles, including methods of proof	• Direct proof • Proof by mathematical induction • Disproof by counterexample • Proof by contradiction • Proof of existence • Proof by construction • Pigeonhole principle • Symmetry principle
c) Solve mathematical problems using problem solving heuristics	• Working backwards • Uncovering pattern and structure • Solving a simpler/similar problem • Considering cases • Restating the problem (e.g. contrapositive)

Mathematical Results

Students may use the following theorems and results without proof. In addition, they may be required to use the ideas in the proofs of these theorems and results to solve other problems.

(i) (AM-GM inequality) For any nonnegative real numbers $x_1, x_2, \ldots, x_n$,

$$\frac{x_1 + x_2 + \ldots + x_n}{n} \geq \sqrt[n]{x_1 x_2 \ldots x_n},$$

where the equality holds if and only if $x_1 = x_2 = \ldots = x_n$.

(ii) (Cauchy-Schwarz inequality) For any real numbers $u_1, u_2, \ldots, u_n$ and $v_1, v_2, \ldots, v_n$,

$$\left(\sum_{i=1}^{n} u_i v_i\right)^2 \leq \left(\sum_{i=1}^{n} u_i^2\right)\left(\sum_{i=1}^{n} v_i^2\right),$$

where the equality holds if there exists a nonzero constant *k* such that $u_i = kv_i$ for all $i = 1, 2, \ldots, n$.

(iii) (Triangle inequality) For any real numbers $x_1, x_2, \ldots, x_n$,

$$|x_1 + x_2 + \ldots + x_n| \leq |x_1| + |x_2| + \ldots + |x_n|,$$

where the equality holds if $x_1, x_2, \ldots, x_n$ are all nonnegative.

(iv) (The Fundamental Theorem of Arithmetic) Every integer $n > 1$ can be expressed as a product of primes in a unique way apart from the order of the prime factors.

(v) There exist infinitely many primes.

(vi) (Division Algorithm) Let *a* be an integer and *b* a positive integer. Then there exists unique integers *q* and *r*, with $0 \leq r < b$, such that $a = bq + r$.

(vii) If *a* and *b* are positive integers, then their greatest common divisor (gcd) is a linear combination of *a* and *b*, that is there exist integers *s* and *t* such that $\gcd(a, b) = sa + tb$.

(viii) If *a* and *b* are positive integers, and there exist integers *s* and *t* such that $sa + tb = 1$, then *a* and *b* are coprime.

Knowledge of the following theorems is not explicitly required: Euclidean algorithm, Chinese remainder theorem, Wilson's Theorem, Fermat's little theorem, and Euler's theorem.

SECTION 3:
PEDAGOGY AND
FORMATIVE ASSESSMENT

Teaching Processes
Phases of Learning
Formative Assessment
Use of Technology

3. PEDAGOGY AND FORMATIVE ASSESSMENT

Teaching Processes

The Pedagogical Practices of The Singapore Teaching Practice (STP) outlines four Teaching Processes that make explicit what teachers reflect on and put into practice before, during and after their interaction with students in all learning contexts.

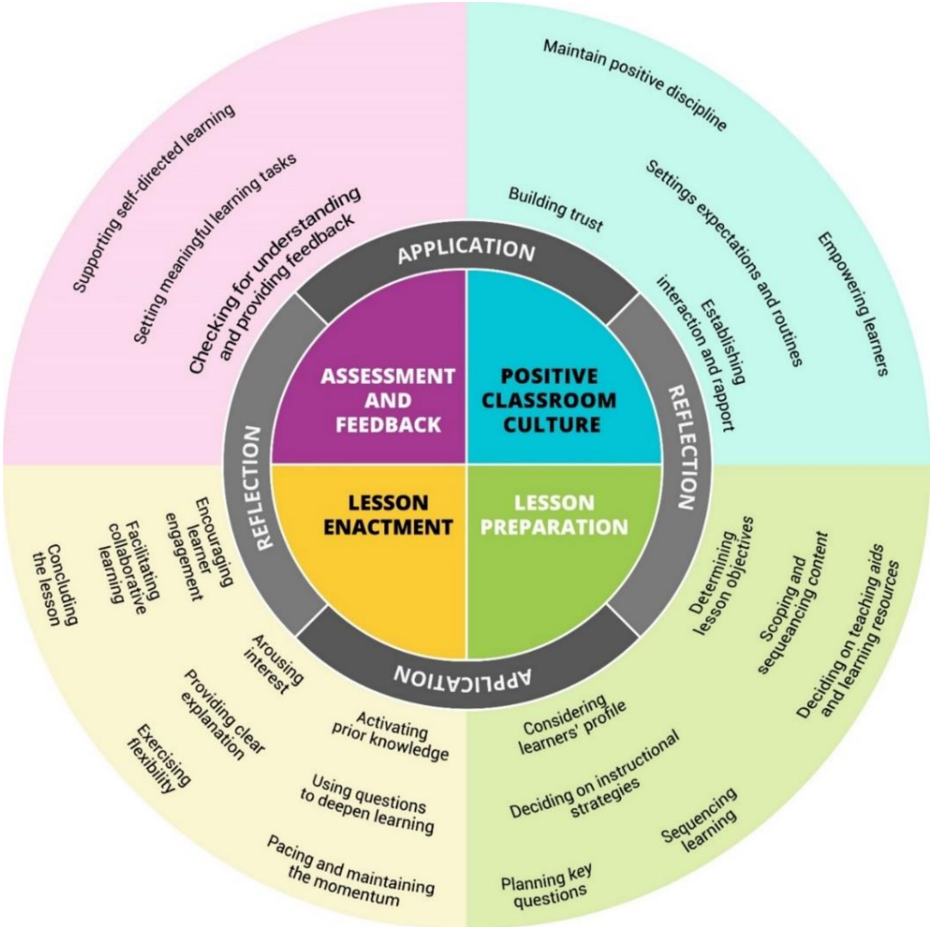
It is important to view the Pedagogical Practices of the STP in the context of the Singapore Curriculum Philosophy (SCP) and Knowledge Bases (KB), and also to understand how all three components work together to support effective teaching and learning.

Taking reference from the SCP, every student is valued as an individual, and they have diverse learning needs and bring with them a wide range of experiences, beliefs, knowledge, and skills. For learning to be effective, there is a need to adapt and match the teaching pace, approaches and assessment practices so that they are developmentally appropriate.

The 4 Teaching Processes are further expanded into Teaching Areas as follows:

Assessment and Feedback • Checking for Understanding and Providing Feedback • Supporting Self-Directed Learning • Setting Meaningful Assignments	Positive Classroom Culture • Establishing Interaction and Rapport • Maintaining Positive Discipline • Setting Expectations and Routines • Building Trust • Empowering Learners
Lesson Enactment • Activating Prior Knowledge • Arousing Interest • Encouraging Learner Engagement • Exercising Flexibility • Providing Clear Explanation • Pacing and Maintaining Momentum • Facilitating Collaborative Learning • Using Questions to Deepen Learning • Concluding the Lesson	Lesson Preparation • Determining Lesson Objectives • Considering Learners' Profile • Selecting and Sequencing Content • Planning Key Questions • Sequencing Learning • Deciding on Instructional Strategies • Deciding on Teaching Aids and Learning Resources

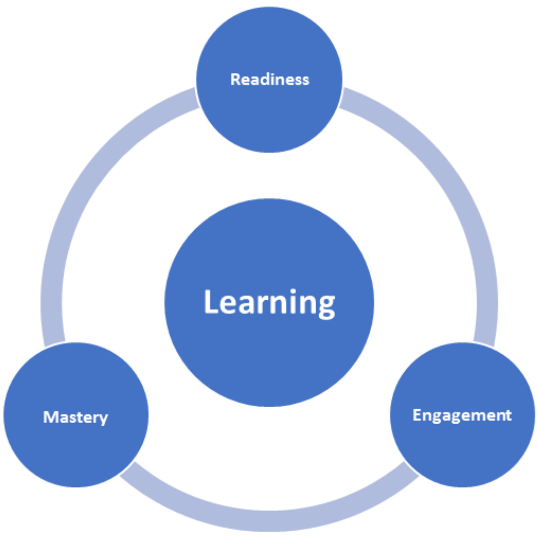
The Teaching Areas are not necessarily specific to a single Teaching Process. Depending on the context, some of the Teaching Areas could be considered in another Teaching Process. The Teaching Processes are undergirded by a constant cycle of application and reflection.



For more information on STP, go to https://www.moe.gov.sg/about/singapore-teaching-practice

Phases of Learning

The Teaching Areas in STP are evident in the effective planning and delivery of the three phases of learning - *readiness*, *engagement* and *mastery*.



Readiness Phase

Student readiness to learn is vital to learning success. Teachers have to consider the following:

- Learning environment
- Students' profile
- Students' prior and pre-requisite knowledge
- Motivating contexts

Engagement Phase

This is the main phase of learning where students engage with the new materials to be learnt (*Encouraging Learner Engagement*). As students have diverse learning needs and bring with them a wide range of experiences, beliefs, knowledge and skills, it is important to consider the pace of the learning and transitions (*Pacing and Maintaining Momentum*) using a repertoire of pedagogies.

Three pedagogical approaches form the spine that supports most of the mathematics instruction in the classroom. They are not mutually exclusive and could be used in different parts of a lesson or unit. Teachers make deliberate choices on the instructional strategies (Deciding on Instructional Strategies) based on learners' profiles and needs, and the nature of the concepts to be taught. The engagement phase can include one or more of the following:

- Activity-based Learning
- Inquiry-based Learning
- Direct Instruction

Regardless of the approach, it is important for teachers to plan ahead, anticipate students' responses, and adapt the lesson accordingly (Exercising Flexibility).

Mastery Phase

The mastery phase is the final phase of learning where students consolidate and extend their learning. To consolidate, teachers summarise and review key learning points at the end of a lesson and make connections with the subsequent lesson (Concluding the Lesson). The mastery phase can include one or more of the following:

- Motivated Practice
- Reflective Review
- Extended Learning

Formative Assessment

Assessment is an integral part of the teaching and learning. It can be formative or summative or both. Formative assessment or Assessment for Learning (AfL) is carried out during teaching and learning to gather evidence and information about students' learning.

The *purpose* of formative assessment is to help students improve their learning and be self-directed in their learning. In learning of mathematics, just as in other subjects, information about students' understanding of the content must be gathered *before*, *during* and *after* the lesson.

The outcomes of the mathematics curriculum go beyond just the recall of mathematical concepts and skills. Since mathematical problem solving is the focus of the mathematics curriculum, assessment should also focus on students' understanding and ability to apply what they know to solve problems. In addition, there should be emphasis on processes such as reasoning, communicating, and modelling.

The overarching objectives of assessment should focus on students':

- understanding of mathematical concepts (going beyond simple recall of facts);
- ability to reason, communicate, and make meaningful connections and integrate ideas across topics;
- ability to formulate, represent and solve problems within mathematics and to interpret mathematical solutions in the context of the problems; and
- ability to develop strategies to solve non-routine problems.

The process of assessment is embedded in the planning of the lessons. The embedding of assessment process may take the following forms:

- Class Activities
- Classroom Discourse
- Individual or Group Tasks

Assessment provides feedback for both students and teachers.

- Feedback from teachers to students informs students where they are in their learning and what they need to do to improve their learning.
- Feedback from students to teachers comes from their responses to the assessment tasks designed by teachers. They provide information to teachers on what they need to do to address learning gaps, how to modify the learning activities students engage in, and how they should improve their instruction.
- Feedback between students is important as well because peer-assessment is useful in promoting active learning. It provides an opportunity for students to learn from each other and also allows them to develop an understanding of what counts as quality work by critiquing their peers' work in relation to a particular learning outcome.

Use of Technology

Computational tools are essential in many branches of mathematics. They support the discovery of mathematical results and applications of mathematics. Mathematicians use computers to solve computationally challenging problems, explore new ideas, form conjectures and prove theorems. Many of the applications of mathematics rely on the availability of computing power to perform operations at high speed and on a large scale. Therefore, integrating technology into the learning of mathematics gives students a glimpse of the tools and practices of mathematicians.

Computational tools are also essential for the learning of mathematics. In particular, they support the understanding of concepts (e.g. simulation and digital manipulatives), their properties (e.g. geometrical properties) and relationships (e.g. algebraic form versus graphical form). More generally, they can be used to carry out investigation (e.g. dynamic geometry software, graphing tools and spreadsheets), communicate ideas (e.g. presentation tools) and collaborate with one another as part of the knowledge building process (e.g. discussion forum). Getting students who have experience with coding to implement some of the algorithms in mathematics (e.g. finding prime factors, multiplying two matrices) can potentially help these students develop a clearer understanding of the algorithms and the underlying mathematics concepts as well.

SECTION 4:
SUMMATIVE ASSESSMENT

Purpose and Assessment Objectives
National Examination

4. SUMMATIVE ASSESSMENT

Purpose and Assessment Objectives

The purpose of summative assessments, such as tests and examinations, is to measure the extent to which students have achieved the learning objectives of the syllabuses.

The assessment objectives reflect the emphases of the syllabus and describe what students should know and be able to do with the concepts and skills learned.

National Examination: H3 Mathematics (Syllabus 9820)

Important information on the national examination for H3 Mathematics is highlighted below. Full details are available on the SEAB website.

The examination will be based on the topics/content listed in Section 2. Knowledge of H2 Mathematics is assumed.

The use of an approved graphing calculator will be expected.

ASSESSMENT OBJECTIVES (AO)
There are three levels of assessment objectives for the examination.

The assessment will test candidates' abilities to:

AO1 Understand and apply mathematical concepts, skills and results to solve non-routine problems, including those that may require integration of concepts and skills from more than one topic.

AO2 Understand and apply advanced methods and techniques of proof to establish the truth or falsity of a mathematical statement.

AO3 Reason and communicate in precise mathematical language through the writing and evaluation of mathematical proofs and solutions.

Notwithstanding the presentation of the topics in the syllabus document, it is envisaged that some examination questions may integrate ideas from more than one topic, and that topics may be tested in the contexts of problem solving and application of mathematics.

SCHEME OF EXAMINATION PAPERS
For the examination in H3 Mathematics, there will be one 3-hour paper marked out of 100. The paper will consist of 8 to 10 questions of different lengths, and each question is worth 9 to 16 marks.

Candidates will be expected to answer all questions.