

# **MATHEMATICS**

## **SYLLABUS**

### **Pre-University**

#### **Higher 2**

#### **Syllabus 9649**

Implementation starting with  
2020 Pre-University One Cohort



Ministry of Education  
SINGAPORE

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# SECTION 1: INTRODUCTION

Nature of Mathematics  
Importance of Learning Mathematics  
Mathematics at the A-Level  
Mathematics Curriculum Framework  
Mathematics and 21CC

# 1. Introduction

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## Nature of Mathematics

Mathematics can be described as a study of the *properties, relationships, operations, algorithms, and applications* of numbers and spaces at the very basic levels, and of abstract objects and concepts at the more advanced levels. Mathematical objects and concepts, and related knowledge and methods, are products of insight, logical reasoning and creative thinking, and are often inspired by problems that seek solutions. *Abstractions* are what make mathematics a powerful tool for solving problems. Mathematics provides within itself a language for *representing* and *communicating* the ideas and results of the discipline.

## Importance of Learning Mathematics

Mathematics contributes to the developments and understanding in many disciplines and provides the foundation for many of today's innovations and tomorrow's solutions. It is used extensively to model and understand real-world phenomena (e.g. consumer preferences, population growth, and disease outbreak), create lifestyle and engineering products (e.g. animated films, mobile games, and autonomous vehicles), improve productivity, decision-making and security (e.g. business analytics, academic research and market survey, encryption, and recognition technologies).

In Singapore, mathematics education plays an important role in equipping every citizen with the necessary knowledge and skills and the capacities to think logically, critically and analytically to participate and strive in the future economy and society. In particular, for future engineers and scientists who are pushing the frontier of technologies, a strong foundation in mathematics is necessary as many of the Smart Nation initiatives that will impact the quality of lives in the future will depend heavily on computational power and mathematical insights.

## Mathematics at the A-Level

There are four syllabuses to cater to the different needs, interests, and abilities of students:

- H1 Mathematics;
- H2 Mathematics;
- H2 Further Mathematics; and
- H3 Mathematics.

**H2 Further Mathematics** is designed for students who are mathematically-inclined and who wish to further expand and deepen their knowledge of mathematics and its applications. Students will develop advanced mathematical thinking and reasoning skills and learn a wider range of mathematical methods and tools. This will give a head-start to students who

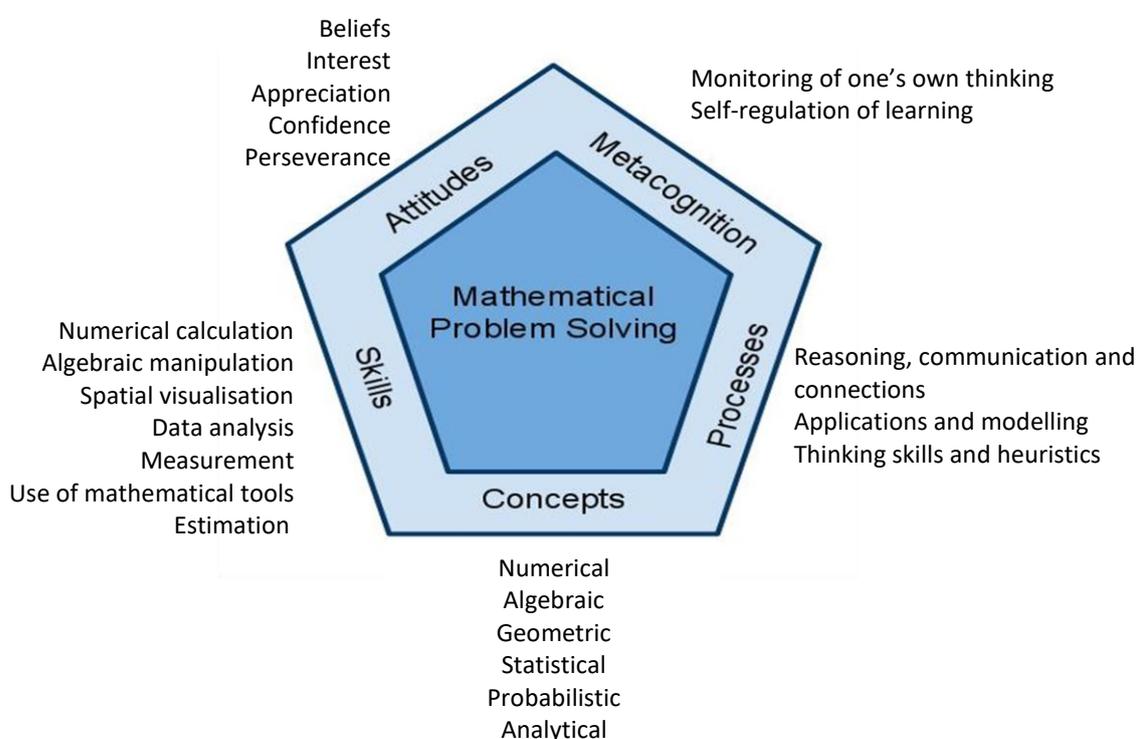
plan to study mathematics or mathematics-related university courses such as science and engineering in the form of a stronger and richer foundation in mathematics.

**H2 Further Mathematics is to be taken with H2 Mathematics as ‘double mathematics’.**

Learning mathematics at the A-Level provides students, regardless of the intended course of study at the university, with a useful set of tools and problem solving skills. It also exposes students to a way of thinking that complements other ways of thinking developed through the other disciplines.

**Mathematics Curriculum Framework**

- *Mathematical Problem Solving*



The central focus of the mathematics curriculum is the development of mathematical problem solving competency. Supporting this focus are five inter-related components – concepts, skills, processes, metacognition and attitudes. The framework sets the direction for and provides guidance in the teaching, learning, and assessment of mathematics.

- *Concepts*

Mathematical concepts can be broadly grouped into *numerical, algebraic, geometric, statistical, probabilistic, and analytical* concepts. These content categories are connected and interdependent. At different stages of learning and in different syllabuses, the breadth and depth of the content vary.

- *Skills*

Mathematical skills refer to *numerical calculation, algebraic manipulation, spatial visualisation, data analysis, measurement, use of mathematical tools, and estimation*. The skills are specific to mathematics and are important in the learning and application of mathematics. In today's classroom, these skills also include the abilities to use spreadsheets and other software to learn and do mathematics.

- *Processes*

Mathematical processes refer to the process skills involved in the process of acquiring and applying mathematical knowledge. These include *reasoning, communication and connections, applications and modelling, and thinking skills and heuristics* that are important in mathematics.

*Reasoning, communication and connections:*

- Mathematical reasoning refers to the ability to analyse mathematical situations and construct logical arguments.
- Communication refers to the ability to use mathematical language to express mathematical ideas and arguments precisely, concisely and logically.
- Connections refer to the ability to see and make linkages among mathematical ideas, between mathematics and other subjects, and between mathematics and the real world.

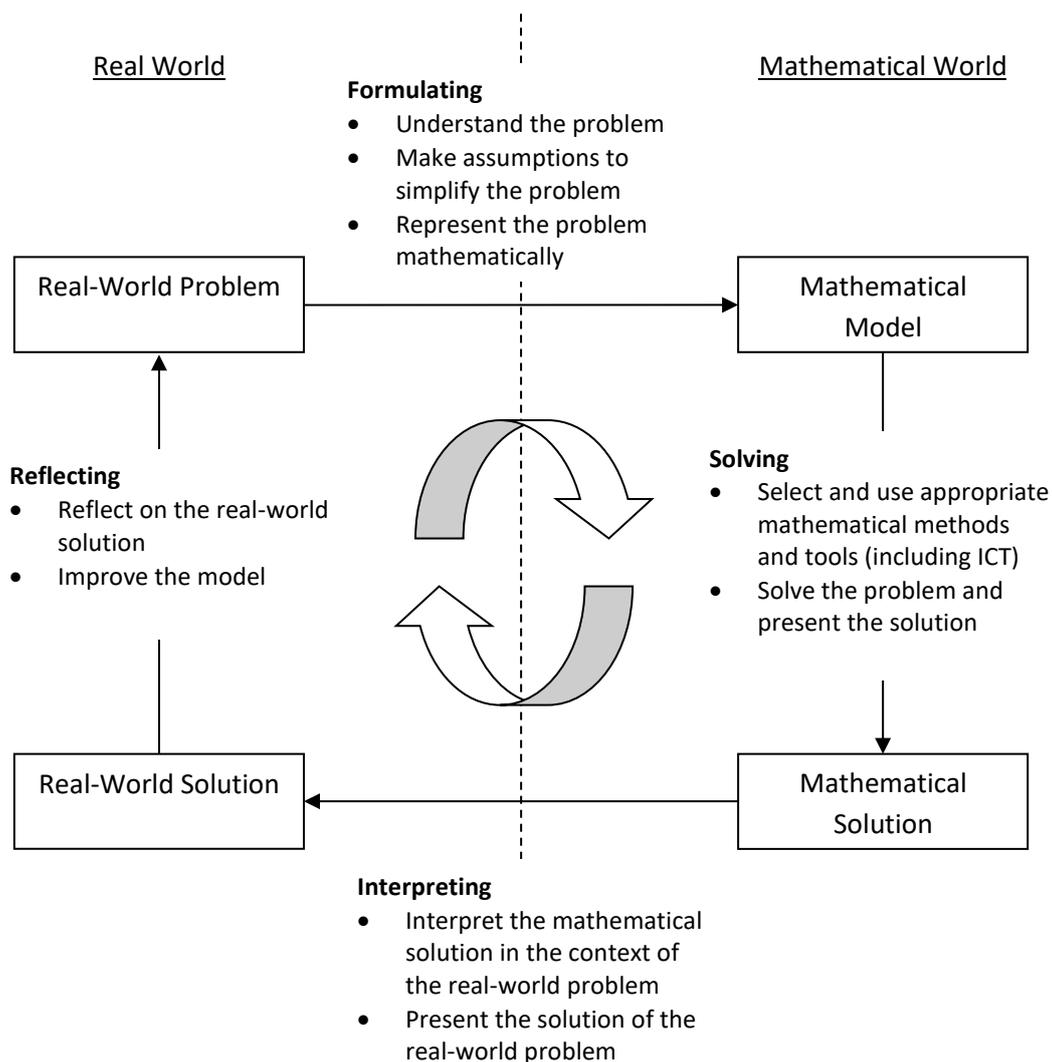
*Applications and modelling* allow students to connect mathematics to the real world, enhance understanding of key mathematical concepts and methods, as well as develop mathematical competencies. Mathematical modelling is the process of formulating and improving a mathematical model<sup>1</sup> to represent and solve real-world problems. Through mathematical modelling, students learn to deal with complexity and ambiguity by simplifying and making reasonable assumptions, select and apply appropriate mathematical concepts and skills that are relevant to the problems, and interpret and evaluate the solutions in the context of the real-world problem. [The mathematical modelling process is shown in the diagram on the following page.]

*Thinking skills and heuristics* are essential for mathematical problem solving. Thinking skills refers to the ability to classify, compare, analyse, identify patterns and relationships, generalise, deduce and visualise. Heuristics are general strategies that students can use to solve non-routine problems. These include using a representation (e.g. drawing a diagram, tabulating), making a guess (e.g. trial and error/ guess and check, making a supposition), walking through the process (e.g. working backwards) and changing the problem (e.g. simplifying the problem, considering special cases).

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<sup>1</sup> A mathematical model is a mathematical representation or idealisation of a real-world situation. It can be as complicated as a system of equations or as simple as a geometrical figure. As the word "model" suggests, it shares characteristics of the real-world situation that it seeks to represent.

## Mathematical Modelling Process



### Metacognition

Metacognition, or thinking about thinking, refers to the awareness of, and the ability to control one's thinking processes, in particular the selection and use of problem-solving strategies. It includes monitoring of one's own thinking, and self-regulation of learning.

- *Attitudes*

Attitudes refer to the affective aspects of mathematics learning such as:

- beliefs about mathematics and its usefulness;
- interest and enjoyment in learning mathematics;
- appreciation of the beauty and power of mathematics;
- confidence in using mathematics; and
- perseverance in solving a problem.

## **Mathematics and 21CC**

Learning of mathematics creates opportunities for students to develop key competencies that are important in the 21st century. As an overarching approach, the A-Level mathematics curriculum supports the development of 21st century competencies (21CC) in the following ways:

1. The content are relevant to the needs of the 21<sup>st</sup> century. They provide the foundation for learning many of the advanced applications of mathematics that are relevant to today's world.
2. The pedagogies create opportunities for students to think critically, reason logically and communicate effectively, working individually as well as in groups, using ICT tools where appropriate in learning and doing mathematics.
3. The problem contexts raise students' awareness of local and global issues around them. For example, problems set around population issues and health issues can help students understand the challenges faced by Singapore and those around the world.

# SECTION 2:

# H2 FURTHER MATHEMATICS SYLLABUS

Preamble  
Aims of Syllabus  
Content Strands  
Applications and Contexts  
Content

## 2. H2 FURTHER MATHEMATICS SYLLABUS (FROM 2020)

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### Preamble

Mathematics drives many of the advancements in sciences, engineering, economics and technology. It is at the heart of many of the innovative products and services today. A strong grounding in mathematics is essential for students who aspire to be scientists, engineers or any other professionals who require mathematical tools to solve complex problems.

H2 Further Mathematics is designed for students who are mathematically-inclined and who intend to specialise in mathematics, sciences or engineering or disciplines with higher demand on mathematical skills. It extends and expands on the range of mathematics and statistics topics in H2 Mathematics and provides these students with a head start in learning a wider range of mathematical methods and tools that are useful for solving more complex problems in mathematics and statistics.

**H2 Further Mathematics is to be offered with H2 Mathematics as a double mathematics course.**

### Syllabus Aims

The aims of H2 Further Mathematics are to enable students to:

- (a) acquire a wider range of mathematical concepts and stronger set of mathematical skills for their tertiary studies in mathematics, sciences, engineering and other related disciplines with a heavier demand on mathematics;
- (b) develop thinking, reasoning, communication and modelling skills through a mathematical approach to problem-solving;
- (c) connect ideas within mathematics and apply mathematics in the contexts of sciences, engineering and other related disciplines; and
- (d) experience and appreciate the rigour and abstraction in the discipline.

### Content Strands

H2 Further Mathematics comprises 3 content strands, namely, *Algebra and Calculus*, *Discrete Mathematics*, *Matrices and Numerical Methods*, and *Probability and Statistics*.

- a) Algebra and calculus play a central role in the understanding, development and applications of many branches of mathematics. The strand adds breadth and depth to the topics taught in H2 Mathematics by broadening and deepening the understanding of important mathematical concepts and opening up a wider range of applications that may be useful for the students. It will include mathematical induction, polar curves, conic sections and additional topics in complex numbers and calculus. Through these topics, students will be exposed to a wider range of

applications in science and engineering, and develop stronger reasoning skills through the writing of mathematical proof.

- b) Discrete Mathematics focuses on discrete structures that have many modern real-world applications, especially in computing. Numerical Methods provide useful tools and algorithms to solve problems where exact solutions are not available. This strand adds breadth by introducing problems of discrete nature, in addition to the continuous ones that require calculus, and an ‘algorithmic approach’ to problem solving in addition to the analytic or algebraic approach that could expose students to basic programming. It will include the study of recurrence relations, matrices and linear spaces and algorithms to solve calculus problems and useful applications such as search engines algorithms.
- c) Probability and Statistics provide the concepts, skills and models to study phenomena where randomness, chance and uncertainty are present. This strand adds breadth and depth to the topics taught in H2 Mathematics by broadening and deepening the understanding of important probability and statistical concepts and offering a larger statistical toolkit that may be useful for future studies and research work. The topics include more statistical and probability models (e.g. general and special continuous random variables such as exponential distribution, additional discrete probability model such as Poisson) and a wider range of applications and statistical methods (e.g. paired sampled tests, non-parametric tests, chi-squared tests) that will be useful in areas as far ranging as genetics and politics.

There are many connections that can be made between the topics within each strand and across strands, even though the syllabus content are organised in strands. These connections should be emphasised as part of teaching and learning, to enable students to integrate the concepts and skills in a coherent manner to solve problems.

### Applications and Contexts

As H2 Further Mathematics is designed for students who intend to specialise in mathematics, science or engineering or disciplines with higher demand on mathematical skills, students should therefore be exposed to the applications of mathematics in science and engineering, so that they can appreciate the value and utility of mathematics in these likely courses of study.

The list illustrates the kinds of contexts that the mathematics learnt in the syllabus may be applied, and is by no means exhaustive.

Applications and contexts	Some possible topics involved
Kinematics and dynamics (e.g. free fall, projectile motion, orbital motion, collisions)	Functions; Calculus; Vectors
Movie graphics	Vectors
Optics (design of mirrors)	Functions; Conic Sections

Optimisation problems (e.g. maximising strength, minimising surface area)	Inequalities; System of linear equations; Calculus
Electrical circuits (including alternating current circuit)	Complex numbers; Calculus
Population growth (e.g. spread of diseases), radioactive decay, heating and cooling problems, mixing, chemical changes, charging	Differential equations
Search engines, cryptography, digital music	Matrices and linear spaces
Financial Maths (e.g. banking, insurance)	Sequences and series; Probability; Sampling distributions
Standardised testing	Normal distribution; Probability
Market research (e.g. consumer preferences, product claims)	Sampling distributions; Hypothesis testing; Correlation and regression
Clinical research (e.g. correlation studies)	Sampling distributions; Hypothesis testing; Correlation and regression
Polling	Confidence intervals; Hypothesis testing
Genetics	Chi-square tests

While students will be exposed to applications and contexts beyond mathematics, they are not expected to learn them in depth. Students should be able to use given information to formulate and solve the problems, applying the relevant concepts and skills and interpret the solution in the context of the problem.

## Content

	Topics/ Sub-topics	Content
<b>SECTION A: PURE MATHEMATICS</b>		
<b>1</b>	<b>Algebra and Calculus</b>	
1.1	Mathematical induction	Include: <ul style="list-style-type: none"> <li>• use of method of mathematical induction to establish a given result involving series and recurrence relations, derivatives, inequalities, or divisibility</li> <li>• formulation of conjectures</li> </ul>
1.2	Complex numbers	Include: <ul style="list-style-type: none"> <li>• geometrical effects of conjugation, addition, subtraction, multiplication and division of complex numbers</li> <li>• loci of simple equations and inequalities such as <math> z - c  \leq r</math>, <math> z - a  =  z - b </math> and <math>\arg(z - a) = \alpha</math> (excluding loci of <math> z - a  = k z - b </math>, where <math>k \neq 1</math> and <math>\arg(z - a) - \arg(z - b) = \alpha</math>)</li> <li>• use of de Moivre's theorem to find the powers and <math>n</math>th roots of a complex number, and to derive trigonometric identities</li> </ul>
1.3	Polar coordinates	Include: <ul style="list-style-type: none"> <li>• simple polar curves (for <math>0 \leq \theta &lt; 2\pi</math> or <math>-\pi &lt; \theta \leq \pi</math> or a subset of either of these intervals; and where <math>r</math> is non-negative throughout the domain)</li> <li>• use of formula <math>A = \frac{1}{2} \int_{\alpha}^{\beta} r^2 d\theta</math> for the area of a sector</li> <li>• arc length of curves defined in polar form</li> </ul>
1.4	Conic sections	Include: <ul style="list-style-type: none"> <li>• equation of conic sections in the cartesian form <math>Ax^2 + By^2 + Cx + Dy + E = 0</math></li> <li>• focus, directrix and eccentricity</li> <li>• geometrical properties of conic sections:               <ul style="list-style-type: none"> <li>- sum of the distance from a point on an ellipse to a pair of fixed points is a constant</li> <li>- difference between distances from a point on a hyperbola to a pair of fixed points is a constant</li> <li>- reflective properties of conic sections</li> </ul> </li> <li>• conic sections in polar form given by <math>r = \frac{ep}{1 \pm e \cos \theta}</math> or <math>r = \frac{ep}{1 \pm e \sin \theta}</math>, where <math>e &gt; 0</math> is the eccentricity and <math> p </math> is the distance between the focus (pole) and the directrix</li> </ul>

	Topics/ Sub-topics	Content
1.5	Applications of definite integrals	Include: <ul style="list-style-type: none"> <li>• arc length of curves defined in cartesian or parametric form</li> <li>• volume of revolution about the <math>x</math>- or <math>y</math>-axis for curves defined in cartesian or parametric form using discs or shells as appropriate</li> <li>• surface area of revolution about the <math>x</math>- or <math>y</math>-axis for curves defined in cartesian or parametric form</li> </ul>
1.6	Differential equations	Include: <ul style="list-style-type: none"> <li>• analytical solution of first order and second order linear differential equations of the form:               <ol style="list-style-type: none"> <li>(i) <math>\frac{dy}{dx} = f(x)g(y)</math></li> <li>(ii) <math>\frac{dy}{dx} + p(x)y = q(x)</math>, using an integrating factor</li> <li>(iii) <math>\frac{d^2y}{dx^2} + a\frac{dy}{dx} + by = 0</math></li> <li>(iv) <math>\frac{d^2y}{dx^2} + a\frac{dy}{dx} + by = f(x)</math>, where <math>f(x)</math> is a polynomial or <math>pe^{kx}</math> or <math>p\cos(kx) + q\sin(kx)</math></li> </ol>               including those that can be reduced to the above by means of a given substitution             </li> <li>• relationship between the solution of a non-homogenous equation and the associated homogenous equation</li> <li>• family of solution curves</li> <li>• exponential growth model</li> <li>• logistic growth model, equilibrium points and their stability, and harvesting</li> </ul> Exclude phase lines, slope fields and bifurcation diagrams.
<b>2</b>	<b>Discrete Mathematics, Matrices and Numerical Methods</b>	
2.1	Recurrence relations	Include: <ul style="list-style-type: none"> <li>• sequence generated by a simple recurrence relation, including the use of graphing calculator to generate the sequence defined by the recurrence relation</li> <li>• behaviour of a sequence, such as the limiting behaviour of a sequence</li> <li>• solution of               <ol style="list-style-type: none"> <li>(i) First order linear (homogeneous and non-homogeneous) recurrence relations with constant coefficients of the form <math>u_n = au_{n-1} + b</math>, <math>a, b \in \mathbb{R}</math>, <math>a \neq 0</math></li> <li>(ii) Second order linear homogeneous recurrence relations with constant coefficients</li> </ol> </li> <li>• modelling with recurrence relations of the forms above</li> </ul>

	Topics/ Sub-topics	Content
2.2	Matrices and linear spaces	<p>Include:</p> <ul style="list-style-type: none"> <li>• use of matrices to represent a set of linear equations</li> <li>• operations on <math>3 \times 3</math> matrices</li> <li>• determinant of a square matrix and inverse of a non-singular matrix (<math>2 \times 2</math> and <math>3 \times 3</math> matrices only)</li> <li>• use of matrices to solve a set of linear equations (including row reduction and echelon forms, and geometrical interpretation of the solution)</li> <li>• linear spaces and subspaces, and the axioms (restricted to spaces of finite dimension over the field of real numbers only)</li> <li>• linear independence and span</li> <li>• basis and dimension (in simple cases), including use of terms such as 'column space', 'row space', 'range space' and 'null space'</li> <li>• rank of a square matrix and relation between rank, dimension of null space and order of the matrix</li> <li>• linear transformations and matrices from <math>\mathbb{R}^n \rightarrow \mathbb{R}^m</math></li> <li>• eigenvalues and eigenvectors of square matrices (<math>2 \times 2</math> and <math>3 \times 3</math> matrices, restricted to cases where the eigenvalues are real and distinct)</li> <li>• diagonalisation of a square matrix <math>M</math> by expressing the matrix in the form <math>\mathbf{QDQ}^{-1}</math>, where <math>\mathbf{D}</math> is a diagonal matrix of eigenvalues and <math>\mathbf{Q}</math> is a matrix whose columns are eigenvectors, and use of this expression such as to find the powers of <math>M</math></li> </ul>
2.3	Numerical methods	<p>Include:</p> <ul style="list-style-type: none"> <li>• location of roots of an equation by simple graphical or numerical methods</li> <li>• approximation of roots of equations using linear interpolation and Newton-Raphson method, including cases where each method fails to converge to the required root</li> <li>• iterations involving recurrence relations of the form <math>x_{n+1} = F(x_n)</math>, including cases where the method fails to converge</li> <li>• approximation of integral of a function using the trapezium rule and Simpson's rule</li> <li>• approximation of solutions of first order differential equations using Euler method (including the use of the improved Euler formula)</li> </ul>

	Topics/ Sub-topics	Content
<b>SECTION B: PROBABILITY AND STATISTICS</b>		
<b>3</b>	<b>Probability and Statistics</b>	
3.1	Discrete random variables	Include: <ul style="list-style-type: none"> <li>• use of Poisson distribution <math>Po(\mu)</math> and geometric distribution <math>Geo(p)</math> as probability models, including conditions under which each distribution is a suitable model</li> <li>• mean and variance for Poisson and geometric distributions</li> <li>• additive property of the Poisson distribution</li> </ul>
3.2	Continuous random variables	Include: <ul style="list-style-type: none"> <li>• probability density function of a continuous random variable and its mean and variance (includes 'piecewise' probability density function)</li> <li>• cumulative distribution function and its relationship with the probability density function</li> <li>• concepts of median and mode of a continuous random variable</li> <li>• use of the result <math>E(g(X)) = \int_{-\infty}^{\infty} g(x)f(x) dx</math> in simple cases, where <math>f(x)</math> is the probability density function of <math>X</math> and <math>g(x)</math> is a function of <math>X</math></li> <li>• uniform distribution and exponential distribution as probability models</li> <li>• relationship between Poisson and exponential distributions</li> </ul>
3.3	Hypothesis testing and Confidence intervals	Include: <ul style="list-style-type: none"> <li>• formulation of hypotheses and testing for a population mean using a small sample drawn from a normal population of unknown variance using a <math>t</math>-test</li> <li>• formulation of hypotheses for the difference of population means, and apply, as appropriate:               <ul style="list-style-type: none"> <li>- a 2-sample <math>t</math>-test</li> <li>- a paired sample <math>t</math>-test</li> <li>- a test using a normal distribution</li> </ul> </li> <li>• contingency tables and <math>\chi^2</math>-tests of:               <ul style="list-style-type: none"> <li>- goodness of fit</li> <li>- independence (excluding Yates' correction for continuity)</li> </ul> </li> <li>• connection between confidence interval and hypothesis test</li> <li>• confidence interval for the population mean based on:               <ul style="list-style-type: none"> <li>- a random sample from a normal population of known variance</li> <li>- a small random sample drawn from a normal population of unknown variance</li> <li>- a large random sample from any population</li> </ul> </li> <li>• confidence interval for population proportion (including</li> </ul>

	Topics/ Sub-topics	Content
		<p>concept of sample proportion) from a large random sample</p> <ul style="list-style-type: none"> <li>• interpretation of confidence intervals and the results of a hypothesis test in the context of the problem</li> </ul> <p>Exclude the use of the term 'Type I error', concept of Type II error and power of a test.</p>
3.4	Non-parametric tests	<p>Include:</p> <ul style="list-style-type: none"> <li>• formulation of hypotheses and testing for: <ul style="list-style-type: none"> <li>- a population median using Sign test</li> <li>- identical probability distributions for two sampled populations in a paired difference design using Wilcoxon matched-pair signed rank test</li> </ul> </li> <li>• advantages and disadvantages of non-parametric tests</li> </ul> <p>Exclude treatment of tied ranks.</p>

# **SECTION 3: PEDAGOGY AND FORMATIVE ASSESSMENT**

Teaching Processes  
Phases of Learning  
Formative Assessment  
Use of Technology

### 3. PEDAGOGY AND FORMATIVE ASSESSMENT

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#### Teaching Processes

The Pedagogical Practices of The Singapore Teaching Practice (STP) outlines four Teaching Processes that make explicit what teachers reflect on and put into practice before, during and after their interaction with students in all learning contexts.

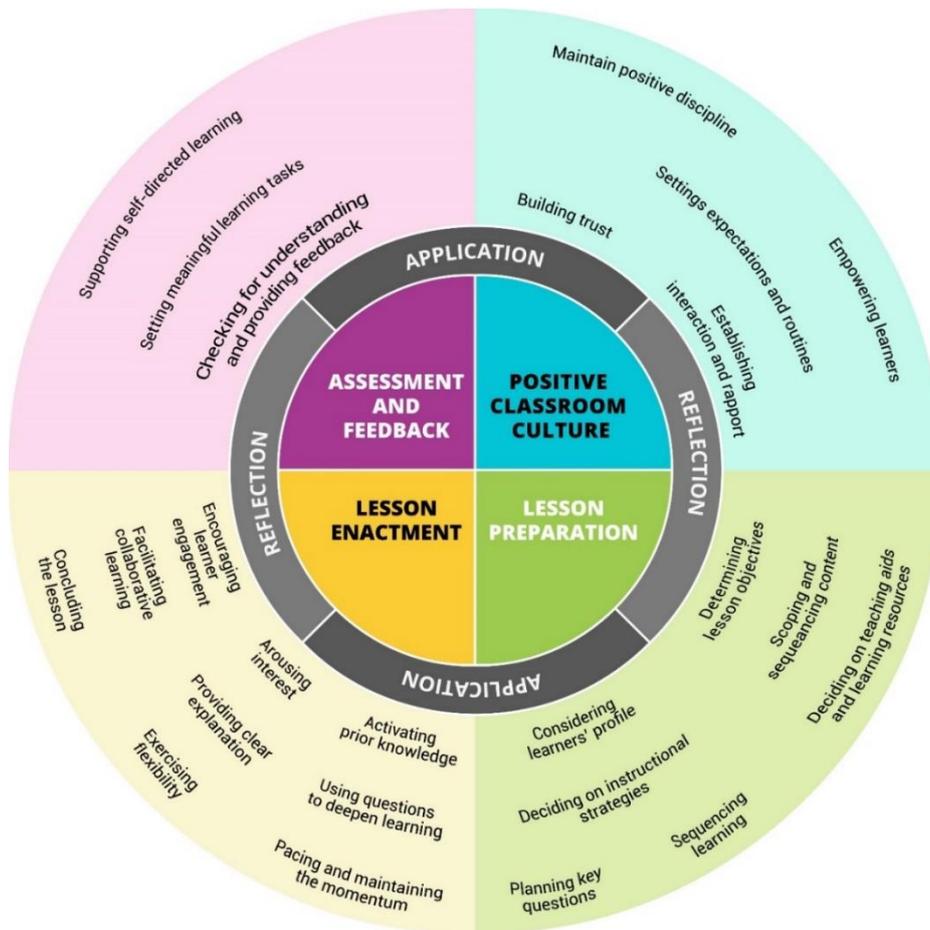
It is important to view the Pedagogical Practices of the STP in the context of the Singapore Curriculum Philosophy (SCP) and Knowledge Bases (KB), and also to understand how all three components work together to support effective teaching and learning.

Taking reference from the SCP, every student is valued as an individual, and they have diverse learning needs and bring with them a wide range of experiences, beliefs, knowledge, and skills. For learning to be effective, there is a need to adapt and match the teaching pace, approaches and assessment practices so that they are developmentally appropriate.

The 4 Teaching Processes are further expanded into Teaching Areas as follows:

<p>Assessment and Feedback</p> <ul style="list-style-type: none"> <li>• Checking for Understanding and Providing Feedback</li> <li>• Supporting Self-Directed Learning</li> <li>• Setting Meaningful Assignments</li> </ul>	<p>Positive Classroom Culture</p> <ul style="list-style-type: none"> <li>• Establishing Interaction and Rapport</li> <li>• Maintaining Positive Discipline</li> <li>• Setting Expectations and Routines</li> <li>• Building Trust</li> <li>• Empowering Learners</li> </ul>
<p>Lesson Enactment</p> <ul style="list-style-type: none"> <li>• Activating Prior Knowledge</li> <li>• Arousing Interest</li> <li>• Encouraging Learner Engagement</li> <li>• Exercising Flexibility</li> <li>• Providing Clear Explanation</li> <li>• Pacing and Maintaining Momentum</li> <li>• Facilitating Collaborative Learning</li> <li>• Using Questions to Deepen Learning</li> <li>• Concluding the Lesson</li> </ul>	<p>Lesson Preparation</p> <ul style="list-style-type: none"> <li>• Determining Lesson Objectives</li> <li>• Considering Learners' Profile</li> <li>• Selecting and Sequencing Content</li> <li>• Planning Key Questions</li> <li>• Sequencing Learning</li> <li>• Deciding on Instructional Strategies</li> <li>• Deciding on Teaching Aids and Learning Resources</li> </ul>

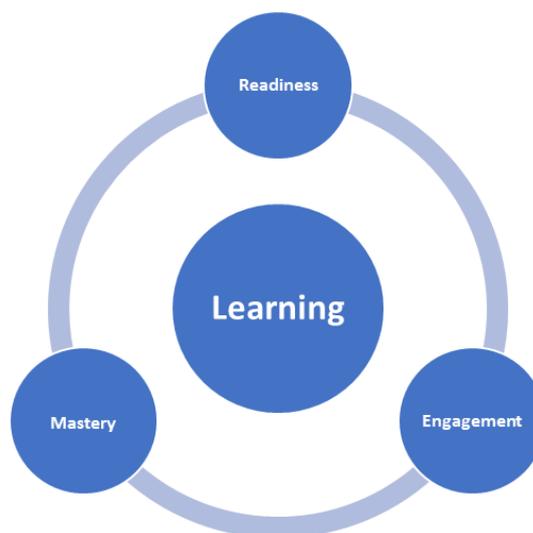
The Teaching Areas are not necessarily specific to a single Teaching Process. Depending on the context, some of the Teaching Areas could be considered in another Teaching Process. The Teaching Processes are undergirded by a constant cycle of application and reflection.



For more information on STP, go to <https://www.moe.gov.sg/about/singapore-teaching-practice>

### Phases of Learning

The Teaching Areas in STP are evident in the effective planning and delivery of the three phases of learning - *readiness, engagement and mastery*.



### Readiness Phase

Student readiness to learn is vital to learning success. Teachers have to consider the following:

- Learning environment
- Students' profile
- Students' prior and pre-requisite knowledge
- Motivating contexts

### Engagement Phase

This is the main phase of learning where students engage with the new materials to be learnt (*Encouraging Learner Engagement*). As students have diverse learning needs and bring with them a wide range of experiences, beliefs, knowledge and skills, it is important to consider the pace of the learning and transitions (*Pacing and Maintaining Momentum*) using a repertoire of pedagogies.

Three pedagogical approaches form the spine that supports most of the mathematics instruction in the classroom. They are not mutually exclusive and could be used in different parts of a lesson or unit. Teachers make deliberate choices on the instructional strategies (*Deciding on Instructional Strategies*) based on learners' profiles and needs, and the nature of the concepts to be taught. The engagement phase can include one or more of the following:

- Activity-based Learning
- Inquiry-based Learning
- Direct Instruction

Regardless of the approach, it is important for teachers to plan ahead, anticipate students' responses, and adapt the lesson accordingly (*Exercising Flexibility*).

### Mastery Phase

The mastery phase is the final phase of learning where students consolidate and extend their learning. To consolidate, teachers summarise and review key learning points at the end of a lesson and make connections with the subsequent lesson (*Concluding the Lesson*). The mastery phase can include one or more of the following:

- Motivated Practice
- Reflective Review
- Extended Learning

## Formative Assessment

Assessment is an integral part of the teaching and learning. It can be formative or summative or both. Formative assessment or Assessment for Learning (AfL) is carried out during teaching and learning to gather evidence and information about students' learning.

The *purpose* of formative assessment is to help students improve their learning and be self-directed in their learning. In learning of mathematics, just as in other subjects, information about students' understanding of the content must be gathered *before, during* and *after* the lesson.

The outcomes of the mathematics curriculum go beyond just the recall of mathematical concepts and skills. Since mathematical problem solving is the focus of the mathematics curriculum, assessment should also focus on students' understanding and ability to apply what they know to solve problems. In addition, there should be emphasis on processes such as reasoning, communicating, and modelling.

The overarching objectives of assessment should focus on students':

- understanding of mathematical concepts (going beyond simple recall of facts);
- ability to reason, communicate, and make meaningful connections and integrate ideas across topics;
- ability to formulate, represent and solve problems within mathematics and to interpret mathematical solutions in the context of the problems; and
- ability to develop strategies to solve non-routine problems.

The process of assessment is embedded in the planning of the lessons. The embedding of assessment process may take the following forms:

- Class Activities
- Classroom Discourse
- Individual or Group Tasks

Assessment provides feedback for both students and teachers.

- Feedback from teachers to students informs students where they are in their learning and what they need to do to improve their learning.
- Feedback from students to teachers comes from their responses to the assessment tasks designed by teachers. They provide information to teachers on what they need to do to address learning gaps, how to modify the learning activities students engage in, and how they should improve their instruction.
- Feedback between students is important as well because peer-assessment is useful in promoting active learning. It provides an opportunity for students to learn from each other and also allows them to develop an understanding of what counts as quality work by critiquing their peers' work in relation to a particular learning outcome.

## Use of Technology

Computational tools are essential in many branches of mathematics. They support the discovery of mathematical results and applications of mathematics. Mathematicians use computers to solve computationally challenging problems, explore new ideas, form conjectures and prove theorems. Many of the applications of mathematics rely on the availability of computing power to perform operations at high speed and on a large scale. Therefore, integrating technology into the learning of mathematics gives students a glimpse of the tools and practices of mathematicians.

Computational tools are also essential for the learning of mathematics. In particular, they support the understanding of concepts (e.g. simulation and digital manipulatives), their properties (e.g. geometrical properties) and relationships (e.g. algebraic form versus graphical form). More generally, they can be used to carry out investigation (e.g. dynamic geometry software, graphing tools and spreadsheets), communicate ideas (e.g. presentation tools) and collaborate with one another as part of the knowledge building process (e.g. discussion forum). Getting students who have experience with coding to implement some of the algorithms in mathematics (e.g. finding prime factors, multiplying two matrices) can potentially help these students develop a clearer understanding of the algorithms and the underlying mathematics concepts as well.

# **SECTION 4:**

# **SUMMATIVE ASSESSMENT**

Purpose and Assessment Objectives  
National Examination (Syllabus 9649)

## 4. SUMMATIVE ASSESSMENT

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### Purpose and Assessment Objectives

The purpose of summative assessments, such as tests and examinations, is to measure the extent to which students have achieved the learning objectives of the syllabuses.

The assessment objectives reflect the emphases of the syllabuses and describe what students should know and be able to do with the concepts and skills learned.

### National Examination: H2 Further Mathematics (Syllabus 9649)

Important information on the national examination for H2 Further Mathematics is highlighted below. Full details are available on the SEAB website.

The examination will be based on the topics/content listed in Section 2. Knowledge of H2 Mathematics is assumed.

The use of an approved graphing calculator will be expected.

### ASSESSMENT OBJECTIVES (AO)

There are three levels of assessment objectives for the examination.

The assessment will test candidates' abilities to:

- AO1** Understand and apply a wide range of mathematical concepts and skills in a variety of problems, including those that may be set in unfamiliar contexts, or require integration of concepts and skills from more than one topic.
- AO2** Formulate real-world problems mathematically, solve the mathematical problems, interpret and evaluate the mathematical solutions in the context of the problems.
- AO3** Reason and communicate mathematically through forming conjectures, making deductions and constructing rigorous mathematical arguments and proofs.

Notwithstanding the presentation of the topics in the syllabus document, it is envisaged that some examination questions may integrate ideas from more than one topic, and that topics may be tested in the contexts of problem solving and application of mathematics. While problems may be set in context, no assumptions will be made about the knowledge of the context. All information will be self-contained within the problem.

## **SCHEME OF EXAMINATION PAPERS**

For the examination in H2 Further Mathematics, there will be two 3-hour papers, each carrying 50% of the total mark, and each marked out of 100, as follows:

### **PAPER 1 (3 hours)**

A paper consisting of 10 to 12 questions of different lengths and marks based on the Pure Mathematics section of the syllabus.

There will be at least two questions on application of Mathematics in real-world contexts, including those from sciences and engineering. Each question will carry at least 12 marks and may require concepts and skills from more than one topic.

Candidates will be expected to answer all questions.

### **PAPER 2 (3 hours)**

A paper consisting of two sections, Sections A and B.

**Section A** (Pure Mathematics – 50 marks) will consist of 5 to 6 questions of different lengths and marks based on the Pure Mathematics section (i.e. Algebra & Calculus, and Discrete Mathematics, Matrices and Numerical Methods) of the syllabus.

**Section B** (Probability and Statistics – 50 marks) will consist of 5 to 6 questions of different lengths and marks based on the Probability and Statistics section of the syllabus.

There will be at least two questions in Section B on application of Mathematics in real-world contexts, including those from sciences and engineering. Each question will carry at least 12 marks and may require concepts and skills from more than one topic.

Candidates will be expected to answer all questions.